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# MULTIPLICATION:
## MULTI-DIGIT OPERATIONS

Just as we could not possibly memorize every number we will need to add or subtract in our lives, we cannot memorize every number we will need to multiply. We need a method for multiplying numbers we do not have memorized.

Men, using the ability God gave them, have developed methods to solve multiplication problems. The equation shown expresses the method most of us have come to associate with multiplication, which we will refer to as the traditional method.

$$\begin{array}{r} {\scriptstyle 1\phantom{2}} \\ 12 \\ \times\ 9 \\ \hline 108 \end{array}$$

Most textbooks spend a great deal of time teaching the mechanics of this method. After all, it is important to know how to multiply multi-digit numbers.

But if we have only learned to manipulate numbers on a piece of paper, we have missed out. We have missed really understanding how this algorithm describes God's creation. We have missed learning to use multiplication in the tasks God has given us to do.

Let us "reveal" multiplication methods together by taking a look at how multiplication methods describe reality and by exploring some different methods and the purpose they serve.

### Methods for Describing Reality

Multiplication methods organize large problems to which we do not know the answer into a number of smaller problems to which we do know the answer. For instance, suppose we needed to multiply 12 x 9 but had only memorized our multiplication facts through 10. We could think of 12 x 9 as the sum of 10 x 9 and 2 x 9, then add the answers together.

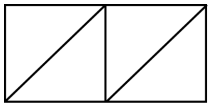
- 10 x 9 = 90
- 2 x 9 = 18
- 12 x 9 = 90 + 18 = 108

Our traditional multiplication method, or algorithm, lets us perform these same steps without having to think about them. Other multiplication methods break problems into different steps or offer different ways to automate the steps.

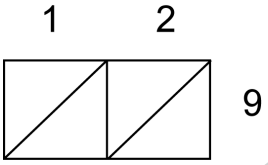
To better see how multiplication methods break up large problems, we will take a look at two different multiplication methods: the gelosia method and the traditional method.

Here are some basic steps to find 9 x 12 using the gelosia method:

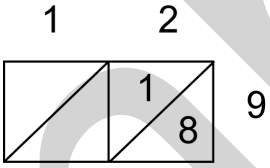
1. Draw several squares and divide those squares into triangles.



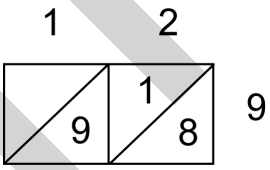
2. Write the two numbers being multiplied above and to the right of the squares.



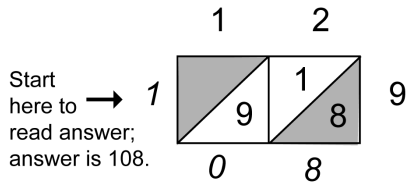

3. Write the answer to 9 x 2 in the first square.



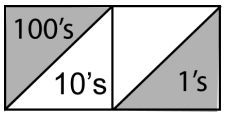

4. Write the answer to 9 x 1 in the second square.




5. Add the numbers in the little triangles diagonally from right to left. In the first diagonal, we have 8, so we write 8 underneath the first triangle. The triangles with the 1 and the 9 are both within the same diagonal, so we add them together and get 10. This gives us our answer: 108. Note: The triangles are shaded in the picture to illustrate which numbers were added together. We wrote the 1 along the left to remind us it was in the hundred's place. We then read the number from left to right to get 108.



Now let us take a closer look at what happened here. In the gelosia method, the squares and triangles separated our ones, tens, and hundreds. When we multiplied 9 x 2, we wrote the 1 part of 18 in the upper triangle because that 1 really represented 1 group of 10. Likewise, when we multiplied 9 x 1, we wrote the 9 in the ten's section of the second square because the 9 really represented 9 tens (the 1 in 12 stands for 1 ten, so when we multiplied 9 x 1, we were really multiplying 9 x 1 ten). When we added up the triangles diagonally, we were really adding up our ones and our tens to find our total answer.




Notice how the gelosia method is a simple way to keep track of place value while we multiplied. Rather than thinking "9 x 2" and "9 x 10," we thought of "9 x 2" and "9 x 1." But the method made sure we wrote the answer to 9 x 1 in the ten's place, since that 1 really stood for 1 ten.

A very similar thing happens in the method with which most of us are familiar. Rather than jumping right to the final way we multiply, let us develop the method step by step by breaking apart the equation 12 x 9 into smaller equations (9 x 2 and 9 x 10) and solving.

```
  12
x  9
  18   (9 x 2) One's column
  90   (9 x 10) Ten's column
 108
```

Now, when we have a task to do over and over again, it is always good to find ways to save steps! There is a way to save steps when we multiply on paper. Instead of writing 18 underneath the line, we could write only the 8 under the line, and put the 1 up on top of the 1 in 12. This reminds us we have 1 ten we will need to add to our ten's column.

```
  1
  12
x  9
   8
```

Now we can work the ten's column. 9 times 1 ten equals 9 tens, or 90, plus the 1 ten we need to add gets us to 10 tens, or 100. If we add 100 and 8, we will get the answer, 108.

```
  1
  12
x  9
   8
 100
 108
```

Now there is still a way to save even more steps. Instead of writing the 100 underneath the 8 and adding it, we could have written 10 next to the 8. Since the 10 is in the ten's column, it stands for 10 groups of 10, or 100.

$$\begin{array}{r} {\scriptstyle 1\phantom{00}} \\ 12 \\ \times\ 9 \\ \hline 108 \end{array}$$

The rules we follow when we multiply keep track of place value, thereby allowing us to break multi-digit multiplication problems we do not have memorized into a series of smaller problems we do have memorized. A multiplication method will only work if it accurately describes the way God causes objects to multiply. If God were not faithfully holding all things together, reducing multiplication to a method would be impossible!

### Many Different Methods

Since multiplication methods describe a real-life consistency, we would expect different people to effectively use different methods. And they do!

Back when written arithmetic methods were first becoming popular in Europe, people experimented extensively with different multiplication methods. I have been continually amazed to discover yet another method or variation on a method. Sometimes, too, the same method had multiple names. The gelosia method, for example, was also called the "quadrilateral, the square, or the method of the cells, and to the Arabs after the 12th century by such names as the method of the sieve or method of the net."[49] People often named a method after whatever they thought it resembled, and sometimes different people chose different names.

Even today, many people use different multiplication methods, some of which are quite different from the typical one taught!

Figure 7 shows just a few of the various methods used throughout history—notice some of them differ only slightly from the method typically taught in math textbooks, and others differ drastically! Note: Appendix D includes an explanation of each of these methods not already covered.

The many different multiplication methods out there remind us that, far from being man-made systems, multiplication methods describe a real-life consistency. Why else would so many different people find methods to arrive at the same answers? Each and every one of these methods ultimately rests on God's faithfulness in holding all things together!

[49] Smith, *History of Mathematics*, 2:114-115. Smith offers several footnoted sources for this statement.

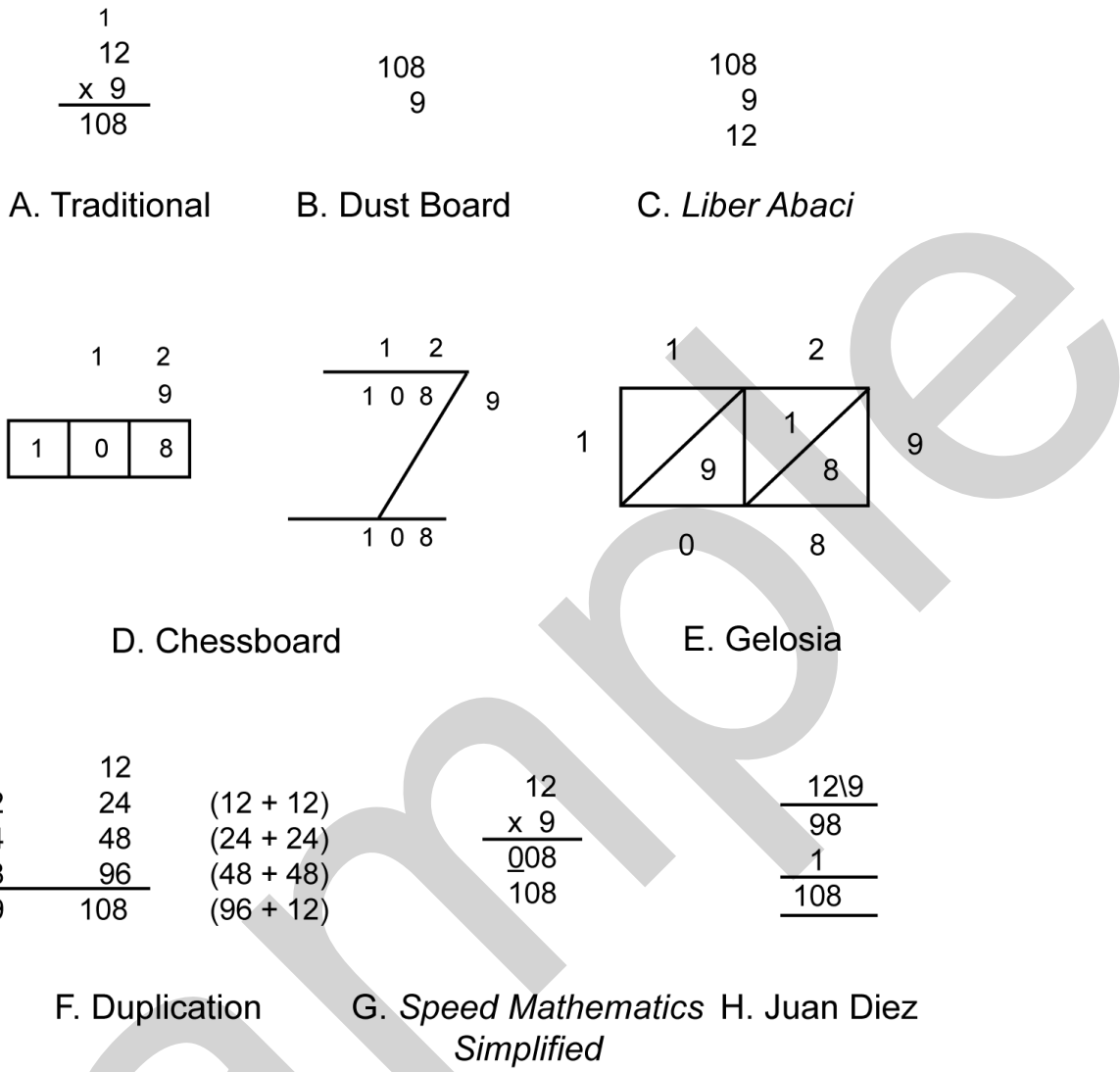



Figure 7: Different Multiplication Methods

**Multiplication - A Useful Tool**

Since multiplication describes a consistent way God governs all things, we should expect multiplication to prove useful in both exploring the world He created and completing the tasks He gives us. And it does!

Have you ever wondered how we know the distance light travels in a year? We cannot physically measure the distance light travels in a year—no measuring tape can extend far enough. We *can*, however, measure the distance light travels in a second. Using multiplication, we can then compute how far light travels in a year!

The section below shows how, starting with the knowledge light travels approximately 186,282 miles a second, we can use multiplication to calculate the distance light travels in a year.[50]

- Seconds in an hour = 60 (seconds in a minute) x 60 (minutes in an hour) = 3,600
- Seconds in a day = 3,600 (seconds in an hour) x 24 (hours in a day) = 86,400
- Seconds in a year = 86,400 (seconds in a day) x 365.25 (days in a year) = 31,557,600
- Distance light travels in a year = 31,557,600 (seconds in a year) x 186,282 miles (distance light travels in a second) = 5,878,612,843,200 miles

We use light-years as a measurement unit to express the vast distances of space. But just because a star is 30,000 light-years from earth does not mean the light took 30,000 years to get here! It only means the star is an incredible distance away! As we just saw, a light-year is just a useful measurement unit based on *the distance we observe light travel in a second here on earth.* Light may very well travel differently in the deep recesses of space, or God may have used some other way to get the light here much faster than normal.[51]

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[50] This example is included here to give you a better understanding of multiplication's purpose. You would not want to share this with your child unless he has dealt with multiple-digit multiplication in the past and is doing this chapter as a review.

[51] A question that often arises when discussing cosmic distances (of stars, galaxies, or other celestial objects) from a biblical perspective is, "How can light from distant stars be seen in a 'young' universe?"

The simple answer is that, in fact, we do not really know. But neither do secular scientists know how light travels. In fact, light-travel time is also one of the big problems for the old-earth Big Bang theory (http://www.answersingenesis.org/creation/v25/i4/lighttravel.asp). The fact is no one fully understands how light, gravity, and other physical forces operate on a cosmic scale.

There are many ways to explain distant starlight. Dr. Russell Humphreys lays out a complete young-earth cosmology, based of the work of Albert Einstein, that neatly solves the distant starlight problem, as well as many other problems secular scientists have been unable to explain (http://creation.com/images/pdfs/cabook/chapter5.pdf). His work has sparked even more research, and there are now several creationist cosmologies (including one by Dr. John Hartnett, author of *Starlight, Time, and the New Physics*) based on "gravitational time dilation." Dr. Jason Lisle, author of *Taking Back Astronomy*, has proposed the "Alternate Synchronization Model," suggesting the universe is divided into "time zones" similar to what we have on Earth (http://www.answersingenesis.org/tj/v15/i1/starlight.asp).

While we do not know exactly how God did it, we do know God created the stars to give light upon the earth. Getting the light here was no problem for our God! "And God set them in the firmament of the heaven to give light upon the earth." Genesis 1:17 (KJV)

For more information on this important topic, please check out the resources on www.answersingenesis.org.

(Thanks to Zak Klein for his help with this footnote.)

Not only does multiplication aid in measuring the distance light travels in a year, it also helps us appreciate God's creation more fully. When we read in a textbook that the *closest* star to earth, Proxima Centauri, is about 4.3 light-years away, it does not sound that far away, does it? But if we use multiplication to convert 4.3 light-years to miles, we will gain a better appreciation for the magnitude of God's universe—even this close star is hardly close!

$$4.3 \times 5{,}878{,}612{,}843{,}200 \text{ miles} \approx 25{,}000{,}000{,}000{,}000 \text{ miles}$$

Imagine driving from Massachusetts to California. According to MapQuest®, the trip is about 3,000 miles and would take more than 40 hours to drive. Using math, we can see this trip is only about 1.2 ten-billionth ($1.2/10{,}000{,}000{,}000$) of the distance to Proxima Centauri!

We can hardly even understand distances like 25,000,000,000,000 miles, and most stars are much, much further away! How much more incredible the God who stretched out the heavens!

> I have made the earth, and created man upon it: I, even my hands, have stretched out the heavens, and all their host have I commanded.
>
> ISAIAH 45:12 (KJV)

> Behold, the nations are as a drop of a bucket, and are counted as the small dust of the balance: behold, he taketh up the isles as a very little thing.
>
> ISAIAH 40:15 (KJV)

How much greater God is than us! Yet this great God knows and cares for each of His children personally.

> I am the good shepherd, and know my sheep, and am known of mine.
>
> JOHN 10:14 (KJV)

By helping us grasp the magnitude of distances like 4.3 light-years, multiplication gives us a better appreciation for the greatness and incomprehensibleness of God, the One who stretched out the distances we cannot even comprehend.

Multiplication also aids in a variety of tasks we face throughout our lives. It has a way of proving useful in the most unlikely places. One spring, our church held a Passover celebration. Everyone attending needed to bring a dish to share. We signed up to bring beef brisket, but we had no idea how much brisket to purchase! Fortunately, the man at the meat counter knew we needed approximately 4 ounces a person. Since we wanted to bring enough for 20 people, we needed about 20 x 4, or 80 ounces, which converts to 5 pounds (80 oz ÷ 16 oz/lb = 5 lb). Since the brisket cost $4.99 a pound, we knew it would cost us about $24.95, since 5 x $4.99 = $24.95.

### Assumptions Matter

Multiplication can only give us valid answers if we multiply the correct numbers and interpret the answers with the correct assumptions. For example, suppose we said, "There are 40 rooms taken at this hotel, and each room can sleep 2 people, so there must be 80 people staying in the hotel." While it is true that 40 x 2 does equal 80, our statement is not necessarily true. It assumes 2 people are sleeping in each room, which is not necessarily true.

This is an important point to grasp. Just because someone tells us they have mathematically proven something does not mean what they have "proven" is necessarily true. The assumptions behind math make a difference. We need to be careful not to place our faith in math itself.

To use multiplication effectively, the appropriate numbers need multiplied! Otherwise, completely correct multiplication could result in an untrue answer.

### Conclusion

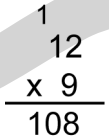
Multiplication can be reduced to a method because of the amazing consistency in the way God causes objects to multiply. Because multiplication describes a real-life consistency, we use it to learn about and explore God's creation, all the while seeing glimpses of God's character. When used appropriately, multiplication serves as a useful tool in the work God has given us to do.

## TEACHING SUGGESTIONS AND IDEAS

**Objective:** *To teach your child to effectively multiply multi-digit numbers with the view of using this knowledge as a practical tool.*

**Specific Points to Communicate:**

- *The multiplication method being taught is one way of simplifying the process of multiplying numbers.*
- *Multiplication can only be reduced to a method because God holds all things together consistently.*
- *Multiplication helps us explore God's creation and aids us in our daily tasks.*




Teach multiplication methods as logical methods rather than rote rules. For example, in the equation 12 x 9, instead of saying, "We carry the 1 to the next column," show your child how the 1 represents 10 objects, so we need to put it over in the column we use to represent groups of 10; carrying is one method for moving numbers representing groups of 10 to the ten's column.

You may find it useful to demonstrate multiplication using a household manipulative (crayons, paper clips, dry beans, etc.), visually demonstrating the consistency the method records. This will both help your child understand the method, and, more importantly, teach him to view math as a tool to describe God's creation instead of as some sort of man-created system.

Later, as your child masters multiplication methods, help him apply them outside a textbook. Teach him to explore the distances to stars, organize events, evaluate options, and much, much more! Practical explorations can go a long way in preparing your child to use math effectively in whatever task or situation the Lord may one day send his way.

**Example**

Teaching math from a biblical worldview means so much more than *saying* over and over again, "God created and sustains a consistent universe and math records it." It is a fascinating journey of actually *showing* this and teaching the child to use each new tool while praising the Creator!

I know most multiplication presentations seem more factual than fascinating. Your child's textbook might read something like the paragraph below, and be filled with apparently meaningless practice problems.

*Multiplying a two-digit number by a one-digit number.*
*Step 1: Multiply 3 times 7 ones.*
*3 sets of 7 ones = 21 ones.*

*Step 2: Rename 21 ones as 2 tens and 1 one.*
*Write a 1 in the ones place.*
*Write a 2 in the tens box...*[52]

Although factual-sounding, these steps are important! These steps explain how to use an important tool. If we were given a special kit to build, we would pour over the instruction manual until we had mastered what we needed to do and could build the kit. Multiplication steps, likewise, help us master a useful tool!

One way you can help your child grasp this is to have him use multiplication outside a textbook. In fact, do not be afraid to break from the textbook altogether and substitute the textbook's problems with real-life situations for your child to solve. Help him learn how to use this tool rather than forcing him to spend hours mastering a method that apparently has no purpose. Children need to see where multiplication leads—how *they* can use it as a tool!

> There can be nothing more destructive of true education than to spend long hours in the acquirement of ideas and methods which lead nowhere.[53]
>
> ALFRED NORTH WHITEHEAD

As you teach multiplication steps, present the thinking behind those steps—show how the steps describe what happens in real life. Most textbooks try to some degree to

[52] Jacobs, et al., *Math 3 for Christian Schools: Teacher's Edition*, 463 (Lesson 129). Quoting from the student booklet, 257.
[53] Alfred North Whitehead, *Essays in Science* (London: Rider and Company, 1948), 133. Quoted in Nickel, *Mathematics: Is God Silent?* 289.

communicate this, but the student does not always get this point. It often gets lost in his attempt to memorize the steps, solve the problems, and finish math for the day.

You can use other multiplication methods to help your child better understand the thinking behind the method he is learning. Looking at other methods can also help your child see multiplication as a method dependent on God and the consistencies He sustains rather than as a man-made or self-existent fact.

Bringing in science can also be quite beneficial. As we use math in science to explore God's creation (such as a light-year), we continually see glimpses of God's character.

Wondering how to about doing all that? Keep reading! The section below offers some hands-on ideas to help you accomplish these goals.

### Ideas

- **Explore other multiplication methods.** People continue to think of different methods to help them multiply! Appendix D offers explanations of a few methods. You may even stumble upon a method your child finds easier! Note: Please use discretion when looking at other methods, as you do not want to cause confusion by switching back and forth between methods.

- **Look for ways to have your child apply multiplication in a real-life example.** See the "Multiplication: Multi-Digit Operations - Multiplication in Real Life" worksheet on page 147 for several examples. Each of these could serve as a launching pad for other similar examples. Keep your eyes open for ways you use multiplication in everyday life, and incorporate them into your math. If you love science, you might want to pull in an application or two of multiplication from science. You could also utilize some word problems from old math books. Back in the 1800s, math books teemed with practical examples. See appendix E for a list of some old math books you can access for free on Google Books. The books are all searchable, and searching a book for "multiplication" should take you to the appropriate sections.

- **Have your child evaluate options for a real-life situation.** We often use multiplication when finding costs and evaluating options. For example, suppose your family needed to spend a few months in an apartment while building a new home. You would need to decide whether to rent a furnished apartment or a non-furnished one. If you opted for a non-furnished apartment, you would need to decide whether you should rent furniture or pay to have some of your furniture delivered. To help you make your decision, you might need to find out how much each of your

options would cost. You could use multiplication to help you!

The "Multiplication: Multi-Digit Operations - Apartment Rental" worksheet on page 148 gives some imaginary data regarding apartments and different options someone in the above situation might face.

Your child will encounter many situations in life where he will want to use multiplication to help him evaluate different options. Teach him how to use math to evaluate options while depending on God and seeking His wisdom rather than relying on his own. While math can help us see the most economical choice, God may have a different choice in mind.

Consider the woman who poured perfume on Jesus' head (Mark 14:3-9). To onlookers, it appeared she completely wasted very expensive perfume. Jesus' disciples wondered why she had not given the money to the poor, but Jesus was pleased by what she had done. In His eyes, "wasting" the perfume was the best decision she could have made.

- **Point out multiplication in more advanced math concepts.** As your child learns other math concepts, he will keep learning more and more uses for multiplication. Multiplication proves handy in finding the area and perimeter of objects, computing interest rates, and much more!

## PARTING NOTE

Always remember, textbooks do not have to be followed as rigid rules! If your child is not grasping a concept, ask God for a different way of explaining it, if you need to hold off on presenting it for a time, or if there is something else God may be trying to show you or your child through the struggle. God makes a wonderful teacher!

> And all thy children shall be taught of the LORD; and great shall be the peace of thy children.
>
> ISAIAH 54:13 (KJV)

## Multiplication: Multi-Digit Operations - Apartment Rental

*Note: This worksheet involves multiplying larger numbers.*

Fill in the following boxes at the bottom of the sheet based on the data given.

**Furnished Apartment**

| | |
|---|---|
| Apartment Rental (Per Month) | $2,000 |
| Furniture Storage (Per Month) | $ 600 |

**Unfurnished Apartment with Rented Furniture**

| | | |
|---|---|---|
| Apartment Rental (Per Month) | | $800 |
| Furniture (Per Month) | | |
| Bedroom | | $200 |
| Living Room | | $100 |
| Kitchen | | $200 |
| One-Time Rental Fee | $100 | |
| Furniture Storage (Per Month) | | $600 |

**Unfurnished Apartment with Delivered Furniture**

| | | |
|---|---|---|
| Apartment Rental (Per Month) | | $800 |
| One-Time Delivery Fee | $2,600 | |

| | Furnished | Unfurnished with Rented Furniture | Unfurnished with Delivered Furniture |
|---|---|---|---|
| Price for First Month | | | |
| Recurring Price After First Month | | | |
| Price for 4 Months | | | |
| Price for 7 Months | | | |
| Price for 12 Months | | | |

## Multiplication: Multi-Digit Operations - Apartment Rental (Continued)

Answer the following questions based on the data found on the previous page:

1. Which option would be the most economical if you end up staying 1 month?

2. What is the difference between a furnished apartment and an unfurnished apartment with rented furniture for 1 month?

3. What is the most economical option for 7 months? How much more expensive are the other options?

***Apartment Rental***

| | Furnished | Unfurnished with Rented Furniture | Unfurnished with Delivered Furniture |
|---|---|---|---|
| Price for First Month | $ 2,600 | $ 2,000 | $ 3,400 |
| Recurring Price After First Month | $ 2, 600 | $ 1,900 | $ 800 |
| Price for 4 Months | $10,400 | $ 7,700 | $ 5,800 |
| Price for 7 Months | $18,200 | $13,400 | $ 8,200 |
| Price for 12 Months | $31,200 | $22,900 | $12,200 |

1. Which option would be the most economical if you end up staying 1 month? **Rented Furniture**

2. What is the difference between a furnished apartment and an unfurnished apartment with rented furniture for 1 month? **$600**

3. What is the most economical option for 7 months? How much more expensive are the other options? **Unfurnished with delivered furniture is the most economical option for 7 months. An unfurnished apartment with rented furniture would be $5,200 more, and getting a furnished apartment would be $10,000 more.**

## DIVISION: MULTI-DIGIT OPERATIONS

**Division in Everyday Life (Version A)**

1. Each book cost $4.
   ($200 cost ÷ 50 books = $4 cost per book)

   They would charge $32.
   (8 x $4 cost = $32)

2. You should read 20 pages each day.
   (80 pages ÷ 4 days = 20 pages)

3. You went 10 miles per gallon.
   (90 miles ÷ 9 gallons = 10 miles per gallon)