

[JR HIGH – YEAR 1 – STUDENT ]

# BOOK 1 PRINCIPLES OF MATHEMATICS

BIBLICAL WORLDVIEW CURRICULUM

[Katherine A. Loop]

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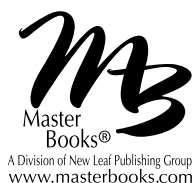
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# **Sample Lesson from Chapter 1**

## Keeping Perspective

We looked today at a few names (one, two, three, etc.) and symbols (1, 2, 3, =, >, <, etc.) used in math. As we continue our study of math, we're going to learn various names and symbols men have adopted to describe different consistencies or operations. Keep in mind that **terms and symbols are like a language** — agreed-upon ways of communicating about the quantities and consistencies around us.

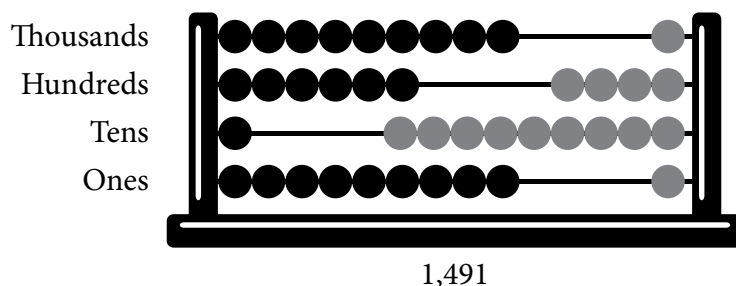
## 1.5 Different Number Systems

It's all too easy to start viewing the terms, symbols, and methods we learn in math as math itself, thereby subtly thinking of math as a man-made system. A look at history, however, reveals many other approaches to representing quantities. Let's take a look at a few of them and at how they compare with our place-value system.

### *Place-Value Systems*

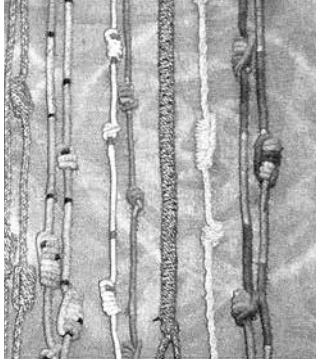
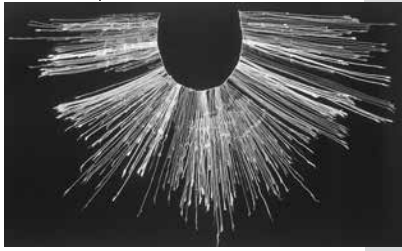
In the last lesson, we reviewed how the number system we're mainly familiar with uses the place, or location, of a digit to determine its value. This is known as a **place-value system**.

Perhaps place value is easiest to picture using a device used extensively throughout the Middle Ages: an abacus. Each bead on the bottom wire of an abacus represents one; on the next, ten; on the third, one hundred; and on the fourth, one thousand. To represent a quantity on an abacus, we move the appropriate number of beads from each wire to the right. In the abacus shown, the 1 bead to the right on the thousands wire represents 1 thousand, the 4 beads to the right on the hundreds wire represent 4 hundred, the 9 beads to the right on the tens wire represent 9 tens, and the 1 bead on the ones wire represent 1. Altogether, that makes 1,491.



Just as the place, or line, of a bead changes its value, the place, or location, of a symbol in a place-value system changes its value. The number system commonly used today is called the **Hindu-Arabic decimal system** (or just the “**decimal system**” for short). This system came from the Hindu system, which the Arabs adopted and brought to Europe.





## The Quipu — An Intriguing Approach

The Incas — an extensive empire in South America spanning more than 15,000 miles — had a fun approach to recording quantities. They tied knots on a device called a quipu (kē pōō).<sup>13</sup> The quipu system was extremely complicated, and only special quipu makers, called quipucamayocs, were able to interpret them. Although we do not know a lot about quipus, we do know they used place value. The location of the knot, along with some other factors, determined its value.

Apparently, the Incas were very successful with this innovative approach. Not only did they operate a huge empire, but the Incas baffled the Spanish conquerors by their ability to record the tiniest details as well as the largest ones on their quipus.<sup>14</sup>

## Fixed-Value Systems

A different approach to recording quantities is to *repeat* symbols to represent other numbers. For example, here are some symbols in Egyptian numerals (hieroglyphic style).<sup>15</sup>

$$\begin{array}{lcl}
 \text{⓪} & = & 1 \\
 \text{Ⓜ} & = & 10 \\
 \text{Ⓢ} & = & 100 \\
 \text{Ⓜ} & = & 1,000
 \end{array}$$

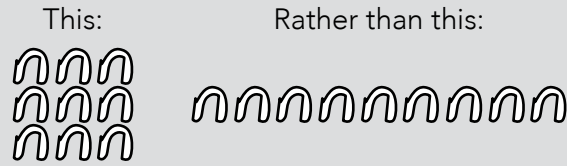
The next figure shows two different quantities represented using Egyptian numerals and our decimal place-value system. Notice how when writing twenty-two, the Egyptians repeated their symbol for one and their symbol for ten twice. They put the smaller values on the left and the larger values on the right. Thus the symbol for ten (Ⓜ) is to the right of the symbol for one (⓪).

Decimal System a place-value system	Egyptian System a fixed-value system
22	⓪⓪ ⓂⓂ
1,491	⓪ ⓂⓂⓂⓂ ⓈⓈⓈⓈ Ⓜ

We'll refer to number systems that use repeated symbols like this as **fixed-value systems**.

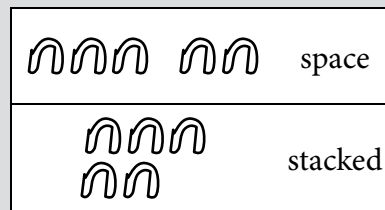
## A Deeper Look at the Egyptian System

Notice that in the Egyptian version of 1,491, the symbols representing “ninety” are stacked on top of each other.

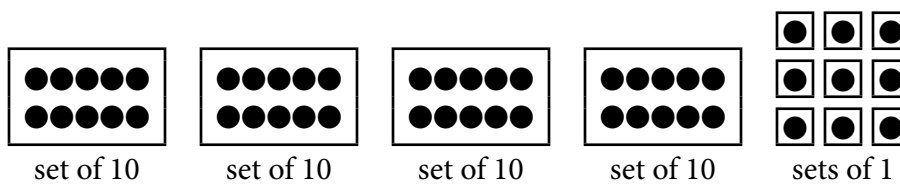


While there were many variations within the Egyptian system over time, in general, when writing more than four of each symbol, the Egyptians **spaced, stacked, or grouped the symbols in sets (groups) of four or less**, with the larger set on top or first.<sup>16</sup> This practice made it easier to count the symbols (and thus to read the number!) at a glance.

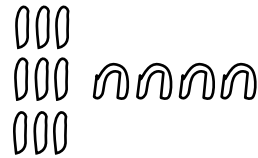
50



Let’s compare our decimal place-value system with the Egyptian system. To record forty-nine objects in the Egyptian system, we would repeat the symbol for “one” nine times to show we had nine ones, and then repeat our symbol for “ten” four times to show we had four sets of ten. In the decimal system, we would instead use our symbols for four and nine, putting the 4 in the tens column so it represents four sets of ten and 9 in the ones column, representing nine sets of one.

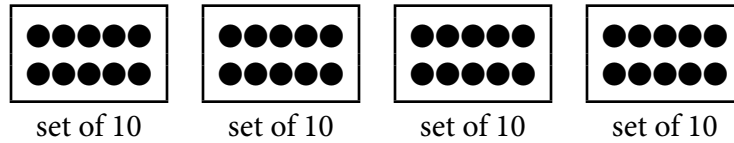


“Forty-nine” = four sets of ten and nine ones

Decimal System a place-value system	Egyptian System a fixed-value system
49	

When we compare forty-nine in both systems, we see it takes significantly fewer symbols to represent the number in the decimal system. Place value saves a lot of extra writing!

To represent a number like forty in the decimal system, we would again use a 4, adding a zero (0) to represent that we have no (0) sets of one. Notice the importance of a zero (0) in a place-value system; without it, we would have no way of showing that the 4 represents 4 sets of ten instead of 4 sets of one.



“Forty” = four sets of ten and no ones

Decimal System a place-value system	Egyptian System a fixed-value system
40	

### Ordered Fixed-Value Systems

Another approach to recording quantities is to again use a limited number of symbols and repeat those symbols, but to add rules regarding their order that change the symbols’ meaning. Roman numerals are an example of an ordered fixed-value system.

Take a look at these symbols used for quantities in Roman numerals:

I	1
V	5
X	10
L	50
C	100
D	500
M	1,000

As with the Egyptians, quantities in Roman numerals are represented by repeating symbols, although this time with the larger quantities on the left.

22 is written XXII in Roman numerals.

But unlike in the Egyptian system, the same symbol is generally not repeated more than three times. Instead, it is assumed that whenever a symbol representing a smaller quantity is to the *left* of a symbol representing a larger quantity, one should *subtract* the value of the smaller quantity from the value of larger quantity to get the value the two symbols represent.

Notice that the smaller quantity is to the *left* of the larger—this means to subtract I from V, giving us  $5 - 1$ , or 4.

Notice that the smaller quantity is to the *right* of the larger—this means to add I to V, giving us  $5 + 1$ , or 6.

I	1	XI	11
II	2	XII	12
III	3	XIII	13
IV	4	XIV	14
V	5	XV	15
VI	6	XVI	16
VII	7	XVII	17
VIII	8	XVIII	18
IX	9	XIX	19
X	10	XX	20

There was a time when “four” was written IIII instead of IV. But IV is easier to read, as there are fewer symbols involved.

Now let’s take a look at the same number we looked at with the Egyptians: 1,491.

Decimal System a place-value system	Roman Numeral System an ordered fixed-value system
1,491	<p><b>MCDXCI</b></p> <p>M = 1,000            CD = 500 – 100 = 400            XC = 100 – 10 = 90            I = 1</p> <p><math>1,000 + 400 + 90 + 1 = 1,491</math></p>

Notice that Roman numerals would not lend themselves well to quickly adding or subtracting on paper! There is a reason we use the decimal place-value system for most purposes.

## Keeping Perspective

While you may use only our current decimal place-value system on a regular basis, being aware of other systems will help you learn to better see our place-value system as just one system to help us describe quantities.

## 1.6 Binary and Hexadecimal Place-value Systems

Before we move on, we’re going to take one more look at the concept of place value, as it’s a pretty important concept. While I’m sure you’re quite familiar with our current place-value system, did you realize computers use place-value systems based on a value besides ten?

Well, they do! They use what’s known as a binary place-value system. Exploring this system, along with the hexadecimal place-value system, is not only cool, but it can also help provide an even firmer grasp of the decimal place-value system. Let’s take a look.

# **Sample Lesson from Chapter 6**

# More with Fractions

## 6.1 Multiplying Fractions

It's time to take a look at multiplying fractions. While you probably already know how to multiply fractions, hopefully our exploration will help you understand better what we're really doing when we multiply fractions and why the method actually works.

### What We're Doing When We Multiply

As we've already explored, multiplication is a way of representing repeated additions — of taking one quantity a certain number of times. Thus,  $3 \times \frac{1}{2}$  means  $\frac{1}{2}$  taken 3 times.




$$\frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{3}{2}, \text{ which equals } 1\frac{1}{2}$$

$$3 \times \frac{1}{2}$$

a shortcut for writing  
the same thing

This can be a little trickier to see when our multiplier is a fraction, such as if we wanted to take  $\frac{1}{2} \times \frac{1}{2}$ . When we have a fraction as our multiplier, we are taking the quantity a partial number of times. What would that mean?

When you think about it, all multiplication can be thought of in terms of *of*. When we multiply  $6 \times 5$ , we're taking 6 sets *of* 5. When we multiply  $3 \times \frac{1}{2}$ , we're taking 3 sets *of*  $\frac{1}{2}$ . Viewing multiplication this way helps make sense out of multiplication when our multiplier is a fraction.



$$\frac{1}{2} \text{ of } \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

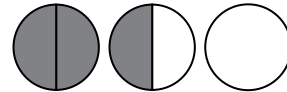
If we take half of a half, we end up with a quarter of a whole.






$$\frac{1}{2} \text{ of } 3 = 1\frac{1}{2}$$

If we take  $\frac{1}{2}$  of 3, we end up with  $1\frac{1}{2}$  wholes.






$$\frac{5}{6} \text{ of } \frac{1}{2} = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

If we take  $\frac{5}{6}$  of a half, we end up with  $\frac{5}{12}$  of a whole.




Notice that when a partial quantity is our multiplier, our product is a smaller quantity instead of a larger. This is because we're taking the quantity a partial number of times.

Remember that multiplication is commutative —  $\frac{1}{2} \times 3$  (i.e.,  $\frac{1}{2}$  of 3) results in the same quantity as  $3 \times \frac{1}{2}$  (i.e., 3 sets of  $\frac{1}{2}$ ).



AND  $3 \times \frac{1}{2} = 1\frac{1}{2}$



$$\frac{1}{2} \times 3 = 1\frac{1}{2}$$

### Getting to a Rule

---

The “rule” for multiplying fractions is to multiply the numerators and the denominators.

$$\frac{5}{6} \times \frac{1}{2} = \frac{5 \times 1}{6 \times 2} = \frac{5}{12}$$

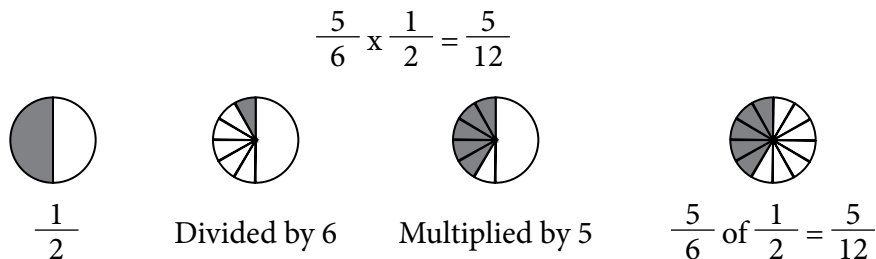
When multiplying a whole number by a fraction, think of the whole number as having 1 as its denominator — after all, dividing by 1 doesn't change the value, and  $\frac{3}{1}$  means  $3 \div 1$ .

$$3 \times \frac{1}{2} = \frac{3}{1} \times \frac{1}{2} = \frac{3 \times 1}{1 \times 2} = \frac{3}{2} = 1\frac{1}{2}$$

We don't necessarily have to bother to rewrite the whole number as a fraction — just remember to view the whole number as a numerator.

While the mathematical proof of why we can multiply fractions by multiplying the numerators and the denominators is beyond the scope of this course, we can see in general that it makes sense. It's obvious to see why it gives us the correct answer in simple cases, such as  $3 \times \frac{1}{2}$ . And  $3 \times 1$  is doing the same thing as adding up the numerators in  $\frac{1}{2} + \frac{1}{2} + \frac{1}{2}$  would. The obvious answer is  $\frac{3}{2}$  which simplifies to  $1\frac{1}{2}$ . But what about  $\frac{5}{6}$  of  $\frac{1}{2}$ ? Why does multiplying by the numerators and denominators here give us the correct answer?

Remember, while we often use fractions to represent partial quantities, they're also a way of writing division. Thus,  $\frac{5}{6}$  means  $5 \div 6$ . To take  $\frac{5}{6}$  of a quantity (i.e., find out what we'd have if we took  $\frac{5}{6}$  of it), we need to divide the quantity by 6 (which we can do by multiplying the denominator by 6), and then take 5 of those parts (which we can do by multiplying the numerator by 5).

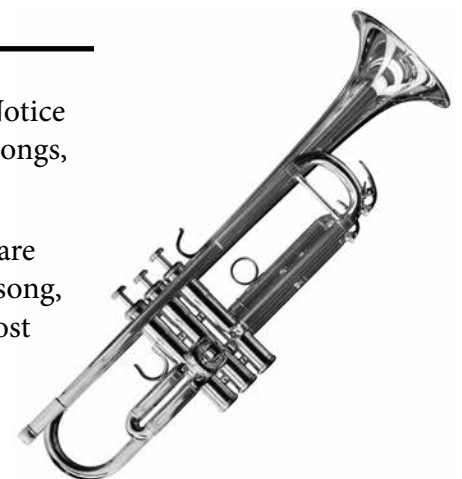


One of the neat things about fractions is that we can view them both as an operation (division) and a quantity (the result of that division). While we pictured fractions as partial quantities in order to understand multiplication better, know that **each of the fractions we looked at was still representing division, too.**

## Fractions in Music





Did you realize that music utilizes fractions? Well, it does! Hum any tune. Notice that we linger on some notes longer than others. When we write music for songs, we need a way of specifying how long to hold each note.

Musicians use different notes to specify different lengths of time. The notes are based on a consistent rhythm called a *beat*. When you clap your hands to a song, you are clapping the beat of the song. The symbol, called a whole note, is most commonly used to specify 4 beats.





A variety of other notes represent a fraction of the whole note. When a whole note is worth 4 beats, a half note is worth 2 beats ( $\frac{1}{2}$  of 4 is 2), a quarter note is worth 1 beat ( $\frac{1}{4}$  of 4 is 1), and an eighth note is worth  $\frac{1}{2}$  of a beat ( $\frac{1}{8}$  of 4 is  $\frac{1}{2}$ ). On the other hand, if the whole note is worth 8 beats, then the half note is worth 4 beats ( $\frac{1}{2}$  of 8 is 4), and so on for the other notes.

			
<b>Whole Note</b>	<b>Half Note</b>	<b>Quarter Note</b>	<b>Eighth Note</b>
1 whole of 4 beats = 4 beats	$\frac{1}{2}$ of 4 beats = 2 beats	$\frac{1}{4}$ of 4 beats = 1 beat	$\frac{1}{8}$ of 4 beats = $\frac{1}{2}$ beat
1 whole of 8 beats = 8 beats	$\frac{1}{2}$ of 8 beats = 4 beats	$\frac{1}{4}$ of 8 beats = 2 beats	$\frac{1}{8}$ of 8 beats = 1 beat

Notice all the times we used the word *of*. You could substitute multiplication for each one.

The application of fractions in music is just one example of how we can use math to help us praise the Lord, serve and encourage others, and simply refresh ourselves.

## Keeping Perspective

Today we looked at multiplication again — at the consistent way even partial quantities multiply. Let's pause for a moment and consider the One holding all this together.

Day in and day out, Jesus is holding everything together so consistently that we can develop methods based on that consistency to work with fractions. Is anything too hard for Him? Of course not! God is perfectly capable of doing everything He says He will do in His Word.

*Behold, I am the LORD, the God of all flesh: is there any thing too hard for me? (Jeremiah 32:27).*

## 6.2 Working with Mixed Numbers

It's time to apply what you've learned about adding, subtracting, and multiplying fractions to mixed numbers. As we do, you'll see that the same principles apply!

### Addition

Say we have  $1\frac{30}{36}$  yards on one bolt, and  $2\frac{9}{36}$  on another. Can you guess how to add these numbers together?

# **Sample Lesson from Chapter 10**

In short, to divide negative numbers, remember to count the negative signs, remembering that each one means *the opposite of*.

This holds true no matter what type of number you're dealing with. Notice that we can divide fractions, decimals, etc., following this same rule.

$$\frac{1}{2} \div -\frac{1}{3} = -\frac{3}{2}$$

*One negative sign — the “opposite of” — gives a negative answer.*

$$-5 \div -0.25 = 20$$

*Two negative signs — the “opposite of the opposite of” — gives a positive answer.*

## Keeping Perspective

Remember: the negative sign indicates *direction*. Each negative sign means *the opposite of*. If you keep this in mind, you'll be able to add, subtract, multiply, and divide negative numbers with ease!

## 10.7 Negative Mixed Numbers

Throughout this chapter, you've been working with negative fractions. It's time now to look at negative mixed numbers. There's nothing really special about them, except that it's easier to get mixed up when dealing with them, so they're worth a special mention.

### Avoiding a Mixed-Number Mix-Up

When working with negative mixed numbers, keep in mind that a negative sign in front of a mixed number applies to the *entire* mixed number.

$$-2\frac{1}{3} \text{ means the same thing as } -2 + -\frac{1}{3}.$$

It's important to remember this when solving problems. Let's take a look at one.

**Example:** Solve  $5\frac{2}{3} - 6\frac{1}{3}$

Let's start by inserting a plus sign for clarity.

$$5\frac{2}{3} + -6\frac{1}{3}$$

One way to add these two mixed numbers would be to add the whole numbers and fractions separately. But we have to be *very* careful about our negative sign with this approach.

Subtracting the whole numbers gives us  $-1$ .

$$5 + -6 = -1$$

Subtracting the fractions gives us  $\frac{1}{3}$ .

$$\frac{2}{3} + -\frac{1}{3} = \frac{1}{3}$$

Now let's put the two together. Since we have a  $-1$  and a  $\frac{1}{3}$ , an easy mistake would be to assume the answer is  $-1\frac{1}{3}$ .

But the  $\frac{1}{3}$  part was *positive*, not negative. We have to add  $+\frac{1}{3}$  to  $-1$ .

$$\frac{1}{3} + -1 = \frac{1}{3} + -\frac{3}{3} = -\frac{2}{3}$$

The correct answer is  $-\frac{2}{3}$ .

Notice how much easier it would be (and how much less chance there is of a mistake with the negative sign) to simply convert the mixed numbers to improper fractions first.

$$5\frac{2}{3} + -6\frac{1}{3}$$

$$\frac{17}{3} + -\frac{19}{3} = -\frac{2}{3}$$

When solving problems with both negative numbers and mixed numbers, you'll avoid mixed-number mix-ups by **first converting the mixed numbers to improper fractions**.

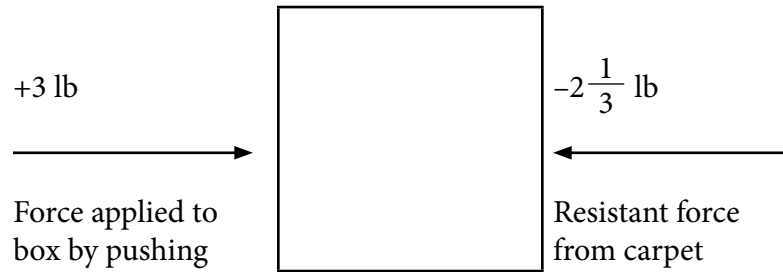
## Keeping Perspective — Negative Numbers and Force

Force is basically the name we give to describe “loosely speaking, a push or pull on the [an] object.”<sup>5</sup> When we throw a ball, our arm gives the ball a certain force that sends it flying through the air. It falls eventually, however, because gravity is also pushing/pulling on it, sending it down toward the earth. Understanding and measuring force helps us work with moving objects, design devices to lift objects, and much more.

Negative numbers play a key role in studying and working with force. We can use negative numbers to describe the force being exerted on an object in one direction, and positive numbers to describe the force being exerted in another direction.

For example, when you try to push a box along a carpeted floor, the carpet is exerting resistance against your push. We could represent the resistance as a negative force, and your push as a positive force. The picture shows the forces in pounds (abbreviated *lb*).





Adding the two forces together will tell you what the resulting force is on the object.

$$+3 \text{ lb} + -2\frac{1}{3} \text{ lb} = +\frac{2}{3} \text{ lb}$$

There's a total of  $\frac{2}{3}$  pound of force in the positive direction.

Force is just one example of how negative numbers help us describe God's creation.

## 10.8 Negative Fractions

So far, we've looked at fractions preceded by negative signs. You know that

$-\frac{1}{2}$  means *the opposite of*  $\frac{1}{2}$ , and how to solve problems with  $-\frac{1}{2}$ .

$$5 + -\frac{1}{2} = 4\frac{1}{2}$$

$$-\frac{1}{2} \cdot 5 = -\frac{5}{2} = -2\frac{1}{2}$$

But what happens if there's a negative sign *within* the fraction? For example, consider  $\frac{-200}{5}$ . Is our answer positive or negative?

To find out, think about what it represents. We could rewrite  $\frac{-200}{5}$  as  $-200 \div 5$ , since fractions are a way of representing division. Then, using what we know about dividing negative numbers, we can tell that the answer would be *negative* ( $-40$ ).

In short, since a fraction represents division, we can figure out if it's negative or positive the **same way we do with division problems** involving negative numbers!

### Handling Negative Signs within a Fraction

$\frac{200}{-5} = -40$	$\frac{-200}{5} = -40$	$\frac{-200}{-5} = 40$
$200 \div -5 = -40$	$-200 \div 5 = -40$	$-200 \div -5 = 40$

# **Sample Lesson from Chapter 15**

## Keeping Perspective

In the previous lesson, we thought through in general how a perimeter related to the sides of a polygon, then specifically to regular polygons and rectangles. In this lesson, we expressed those relationships using symbols, making it super easy to just plug the appropriate numbers in to find the perimeter.

Formulas express consistent relationships. They prove useful because of the consistencies all around us. Ultimately, the usefulness of formulas remind us that we live in a consistent universe and serve a consistent, all-powerful God who holds all things together by the power of His Word!

## 15.3 Area: Rectangles and Squares

Let's continue our exploration of shapes by looking at what we call the **area**, or the space a two-dimensional shape encloses. We'll start our explorations by focusing on rectangles and squares, and then later expand to other shapes.

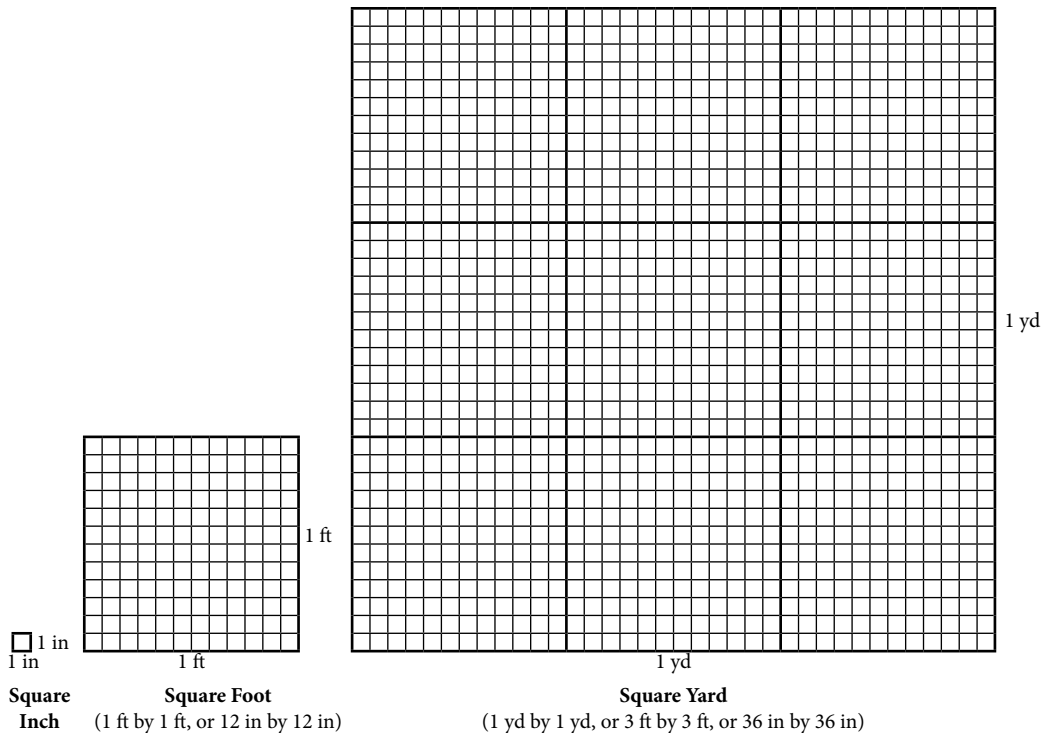
I know you may already know how to find the area of certain shapes, but let's think through the process together. If you're wondering *when* you'll ever need to find an area, just suppose you need to recarpet your bedroom. You'd need to know the area of the room to figure out how much carpet you need.

### *Units for Area*

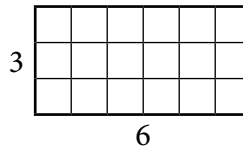
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Before we look at how to find the area, we need a unit to measure it in. The distance units only measure distance in a single direction, whereas area is expressed in two-dimensional space. So we use what we call a **square unit** to measure area. A square inch is 1 inch wide by 1 inch long (in other words, a square with 1-inch sides), a square foot is 1 foot long by 1 foot wide, a square yard is 1 yard long by 1 yard wide — you get the idea.





Take a look at the following picture. Notice how we broke up the space inside, or area, of the rectangle into square units. How many squares fit inside the rectangle?



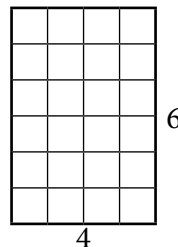
We find that 18 square units fit inside the rectangle above. If each square represented 1 foot wide by 1 foot long, we would say this rectangle has an area of 18 *square feet*. Notice that we use the word “square feet” — the 18 represents 18 1-foot by 1-foot squares.

### ***Finding the Area of a Rectangle***

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Now that we have a way of expressing area, let’s take a look at how to find the area of a rectangle.

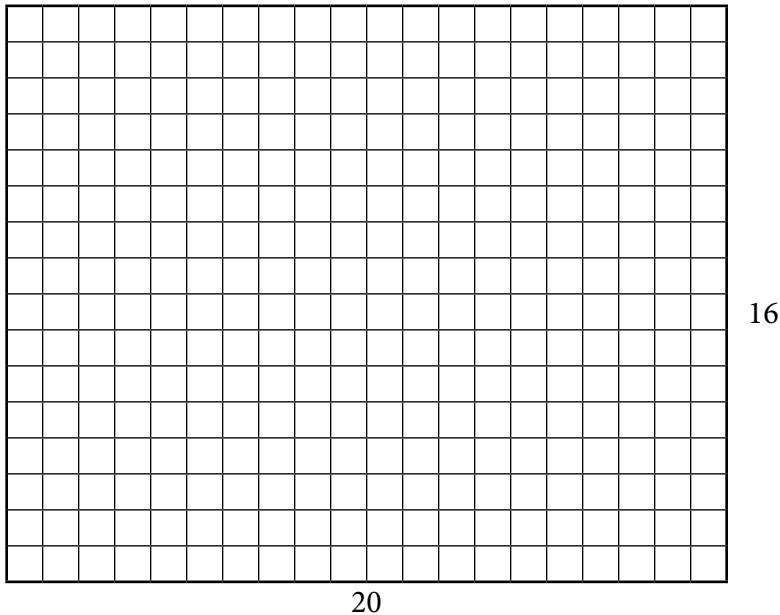
Let’s say we had a rectangle with a length of 4 units and a height of 6 units. What would the area, or space inside, be?





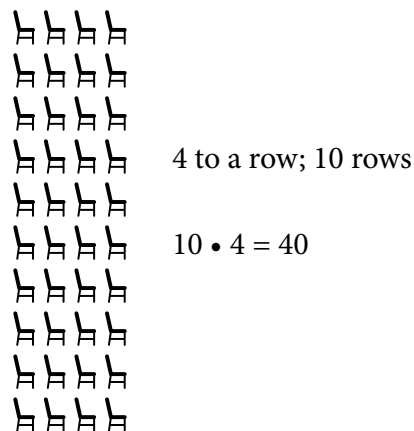
Find out for yourself by counting the squares. You should count 24. This rectangle has an area of 24 square units. If I were to tell you that each unit is 1 square *foot*, you would know that there are 24 square *feet*. However, if each unit is 1 square *inch*, you would have 24 square *inches*.

What would be the area of a rectangle 20 feet long by 16 feet wide/high?



Can you think of a way to find the square units without having to count them? Notice how the length (20) tells us the number of squares in each row, and the width (16) counts the number of rows. So if we **multiply the length by the width**, we'll have counted all the squares! The area of this rectangle is 320 square units, as  $20 \cdot 16 = 320$ .

Notice how this is the same as using multiplication to quickly count any repeated rows — if we lined up chairs so there were 4 to a row and 10 rows, we could add these up by multiplying  $10 \cdot 4$ .



To make things easier, let's represent the area of a rectangle using a formula.

**Area of a Rectangle**

$$\text{Area} = \text{length} \cdot \text{width} \quad \text{or} \quad A = l \cdot w \quad \text{or} \quad A = l(w) \quad \text{or} \quad A = lw$$

$$A = \text{Area}$$

$$l = \text{length}$$

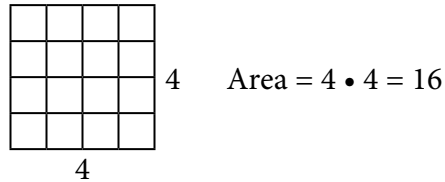
$$w = \text{width (or height)}$$

In other words, **the area of a rectangle equals its length times its width.** It's important to note that sometimes the width of a rectangle is referred to as the height. It doesn't really matter whether you think of the side of a rectangle as the width or the height — the important thing is to know that the area is found by multiplying the two sides together.

**Finding the Area of a Square**

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Since squares are just rectangles (and parallelograms) with all equal sides, we'll be able to find their area by multiplying the length times the width too.



However, since the length and the width are the same, it would be nice to show that all we really have to do is multiply one side by itself.

**Area of a Square**

$$\text{Area} = \text{side} \cdot \text{side} \quad \text{or} \quad A = s \cdot s \quad \text{or} \quad A = s(s)$$

$$A = \text{Area}$$

$$s = \text{side}$$

**Application Time**

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**Example:** Find the area of a rectangle 10 feet long by 3 feet wide.

We know that the area equals the length times the width ( $A = l \cdot w$ ), so all we have to do is plug in the numbers and multiply!

$$A = 10 \text{ ft} \cdot 3 \text{ ft} = 30 \text{ square feet or } 30 \text{ sq ft}$$

Notice that we used square feet to show that the 30 is not just representing a single distance, but rather an *area* measured in 1-foot by 1-foot squares.

**Example:** Find the area of a square with 5-foot sides.

We know the area of a square equals the length of a side times itself ( $A = s \cdot s$ ), so all we have to do is plug in the length of our side and multiply!

$$A = s \cdot s$$

$$A = 5 \text{ ft} \cdot 5 \text{ ft} = 25 \text{ square feet or } 25 \text{ sq ft}$$

## Keeping Perspective — Jesus Used Math Too!

Since we're learning about measurement, I want to quickly point out that carpenters use measurement and math, and Jesus was a carpenter before He began His public ministry (Mark 6:2–3). Think about that for a minute. The Creator and Sustainer of the universe (including math!) humbled Himself to learn a trade, work with his hands, and likely use math in various ways. Jesus knows what it's like to be a man because He became one — fully man and yet fully God. He totally understands every temptation you face — and He can support you through your temptations.

*Wherefore in all things it behoved him to be made like unto his brethren, that he might be a merciful and faithful high priest in things pertaining to God, to make reconciliation for the sins of the people. For in that he himself hath suffered being tempted, he is able to succour them that are tempted (Hebrews 2:17–18).*

*For we have not an high priest which cannot be touched with the feeling of our infirmities; but was in all points tempted like as we are, yet without sin. Let us therefore come boldly unto the throne of grace, that we may obtain mercy, and find grace to help in time of need (Hebrews 4:15–16).*

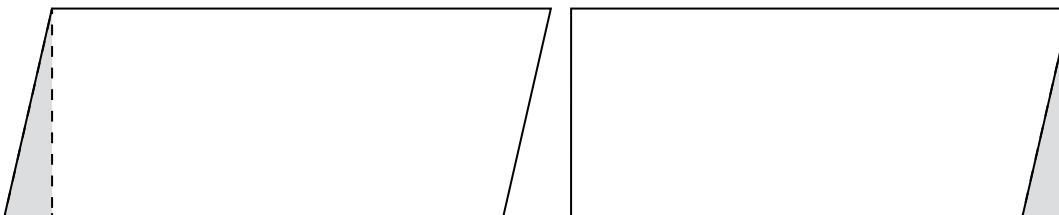
## 15.4 Area: Parallelograms

Let's expand the information we looked at regarding finding the area of rectangles to parallelograms as a whole.

A rectangle is the name we use to describe a specific type of parallelogram. What if instead of a rectangle, we have the parallelogram below? How do we find its area?



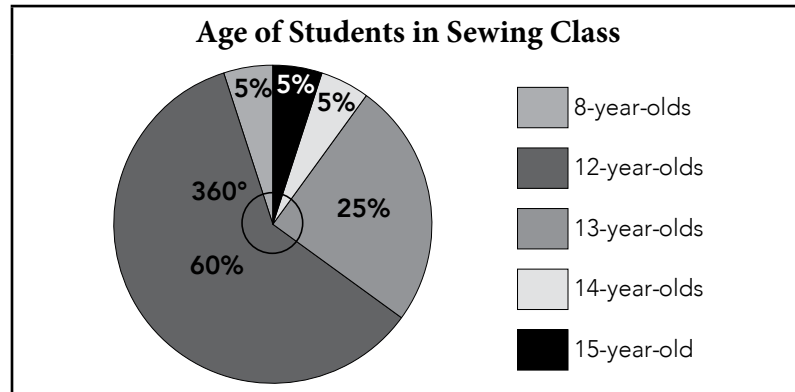
Notice that the parallelogram could be easily rearranged into a rectangle by moving the gray portion to the other side, as shown.



## **Sample Lessons from Chapter 19**

Notice that the sum, or total, of all the angles in the circle equals  $360^\circ$  – this will always be the case.

$$216^\circ + 90^\circ + 18^\circ + 18^\circ + 18^\circ = 360^\circ$$

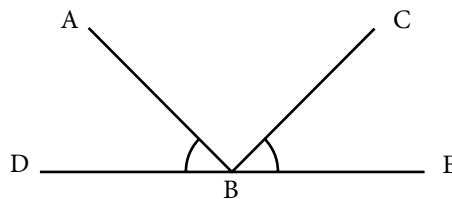


### Keeping Perspective

Notice how drawing a pie graph uses angles! In fact, it requires combining a number of different concepts (percents, multiplication, angles, etc.). It's another example of how the different concepts you learn can be combined to accomplish a task. In the case of a pie graph, our "tools" helped us pictorially represent data.

## 19.4 Expanding Beyond

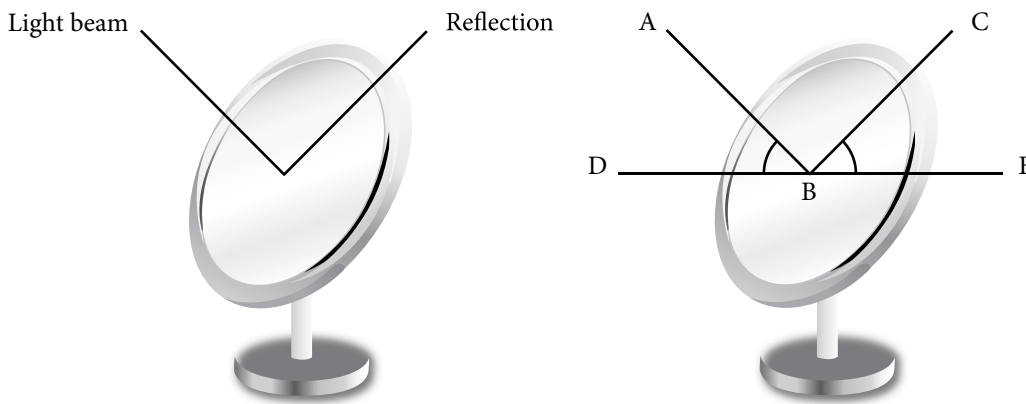
As you move on to high school geometry in a few years, you'll likely encounter a lot of apparently meaningless drawings such as the one shown. And you might be asked to spend a lot of time exploring those drawings and the angles and lines in them.



Always remember those drawings could be used to describe God's creation . . . and that goes beyond shapes! Up until now, our explorations in geometry have focused on exploring actual shapes, but the skills you've been learning help us describe many other aspects of God's creation too.

## Representing Light

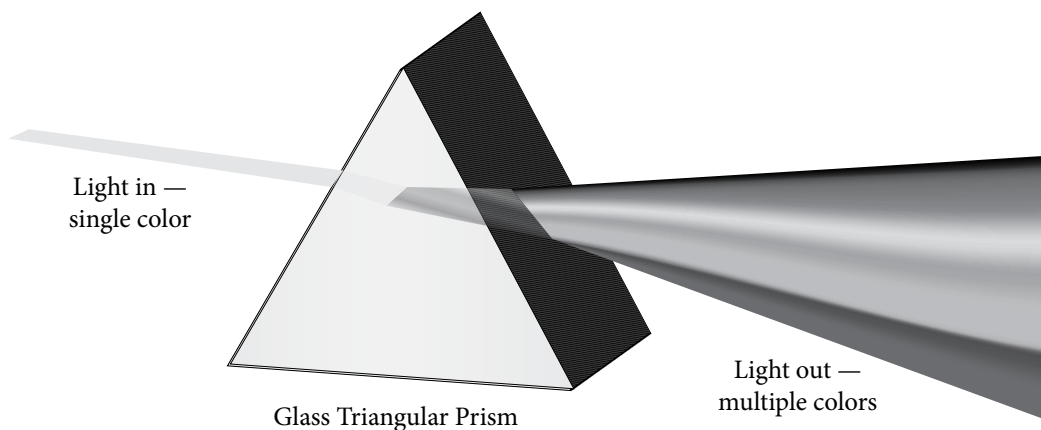
The apparently meaningless drawing shown could actually represent how light reflects off surfaces! Did you realize that God designed light so that when it reflects off a surface, light both hits the surface and reflects off it at the same angle? (Angles in action!)



Not all surfaces reflect light, but when light does reflect, it follows this property.

Knowing this consistency has played a role in designing different technologies, such as CD and DVD players, devices that work based on shining a light and having it reflect at the same angle. That's right — it's the amazing consistency God created and sustains within the very light waves around us that makes DVDs possible.

Have you ever wondered why a glass prism makes rainbows of color? When light strikes something transparent, most of it shines through the object rather than reflecting off of it. But the transparent object changes the angle of the light. In some objects, such as glass triangular prisms, diamonds, and the crystals on most chandeliers, the angles in the transparent object cause the different colors within light to come out the other side at different angles. We then see them as separate colors instead of as one. God has hidden a kaleidoscope of color in every beam of sunlight.



## Application Thought

God created and sustains the light all around us. He understands all the intricacies of its workings. He also understands the intricacies and workings in our lives. We can take comfort knowing God is infinitely wiser than we are. His thoughts and ways are better than ours.

*He hath made the earth by his power, he hath established the world by his wisdom, and hath stretched out the heavens by his discretion (Jeremiah 10:12).*

*Where is the way where light dwelleth? and as for darkness, where is the place thereof, That thou shouldest take it to the bound thereof, and that thou shouldest know the paths to the house thereof? (Job 38:19–20).*

*For as the heavens are higher than the earth, so are my ways higher than your ways, and my thoughts than your thoughts (Isaiah 55:9).*

## Navigating a Ship

Have you ever wondered how sailors navigated across the ocean before the days of the GPS? How did they know their location and where to steer the ship? They used math . . . including angles! Angles are used extensively in navigation.

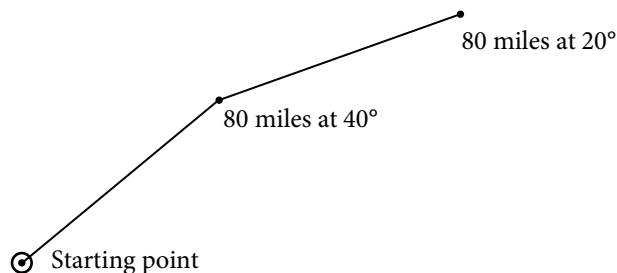
For example, notice the degrees on a compass — a common navigational tool. Degrees are used to refer to directions.

One method of navigation involves measuring the angle between the horizon and a celestial body (such as the sun or a star) and using that information, coupled with charts based off historical data about the sun or star’s position, to figure out one’s location.

Another useful technique known as “dead reckoning” involves angles as well. While we won’t go into the details of this technique, the basic concept isn’t hard to understand. If a ship starts at an angle of  $40^\circ$  and travels 80 miles at that angle before making a turn and traveling at  $20^\circ$  for another 80 miles, the ship’s current location (or rather what it would be if there had been no “leeway, current, helmsman error;”<sup>6</sup> etc.) can be determined.



Compass



## Keeping Perspective

The point? Angles help us describe many different aspects of God's creation. And, as we saw briefly when we looked at the angles in light, as we explore His creation, we end up seeing to a deeper level the wisdom and care with which God created this universe.

## 19.5 Chapter Synopsis and Faulty Assumptions

We've covered a lot of ground in this chapter! Hopefully you caught a glimpse of how angles apply in many more places than you might think.

Here's a quick recap of some of the mechanics:

- We can use units called **degrees** to measure angles. A degree is  $\frac{1}{360^{\text{th}}}$  of a circle.
- **Protractors** are tools with degree markings we can use to easily measure and draw angles.
- A **right angle** is  $90^\circ$ , an **obtuse angle** is greater than  $90^\circ$  but less than  $180^\circ$ , an **acute angle** is less than  $90^\circ$  but greater than  $0^\circ$ , and a **straight line** can be thought of as an  $180^\circ$  angle, also called a **straight angle**.
- Exploring the angles within shapes and between lines proves quite useful and helps us explore real-life shapes . . . as well as other aspects of God's creation that we might not at first think of using geometry to describe (such as a beam of light).

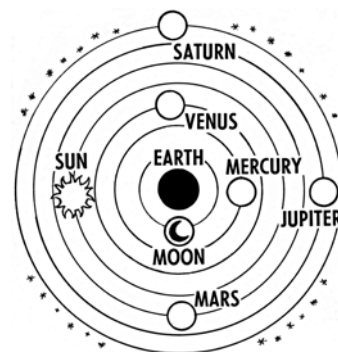
As we continue our study of geometry, we'll keep discovering ways that we can use knowledge of angles, lines, and shapes to help us find other pieces of information. And you'll continue to encounter angles and the need to measure and draw them!

### Ptolemy, the Sun, and the Medieval Ages

Earlier in this chapter, we briefly mentioned that the Greek mathematician Ptolemy broke down the circle into degrees, minutes, and seconds in his work *Syntaxis*. What we didn't have time to discuss then was that *Syntaxis* also laid out a mathematical theory that the sun circled the earth.<sup>7</sup> This theory then became accepted as fact for centuries — so much so that the Medieval Church, not wanting to contradict math and science, tried to add this teaching to the Bible.

Why didn't people question that the sun circled the earth? Ptolemy's math was accurate. Math and reasoning were seen as absolute. Thus, how could the conclusion be wrong?

It wasn't until men like Copernicus (famous for proposing that the earth circled the sun) and Kepler (who discovered the laws of planetary motion) began to





approach math as a *tool* (rather than as an unquestionable source of truth) that old ideas were shown to be false. Using math while observing the world around them, these men questioned the age-old teaching and discovered the truth about planetary orbits — a truth that accurately placed the sun, not the earth, at the center.

Always remember that sound math does not ensure a sound conclusion. If you start from an incorrect assumption, you can make a logical case for a very incorrect conclusion. Math and science are *tools*; they are *not* sources of truth.

The Medieval Church made the mistake of taking man's reasoning as true and finding a way to add it to the Bible.<sup>8</sup> Their mistake caused them lots of embarrassment when it was proven wrong.

Sadly, this backward way of approaching Scripture still goes on today in different forms. For example, many have tried to reinterpret Genesis in order to match Darwinian evolution rather than starting with Scripture and using it to interpret what we find in creation. Don't ever feel that you have to reinterpret Scripture to match what others say. Scripture is true and trustworthy, while man's opinion and even mathematical proofs are subject to error and revising.

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For more information  
on how the literal  
Genesis account does  
indeed match the world  
in which we live, see  
[AnswersinGenesis.org](http://AnswersinGenesis.org).  
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